Arithmetic with Fractions

Ah, arithmetic with fractions. That's what calculators are for, right? Except most calculators can't handle something like $\frac{x}{y^2} + \frac{2ab}{\pi}$. And even if they can, we still should be able to do it ourselves (just like even though you might have a GPS on your phone, that's not excuse for not knowing which way is up on a map). We'll quickly review the rules here (*very quickly*), but the best way to get these down pat is to practice, practice, practice.

1. Adding and Subtracting Fractions

Adding fractions is a pain. You have to find a common denominator. But then it's easy, just add the numerators. For example,

$$\frac{1}{2} + \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{3} + \frac{1}{3} \cdot \frac{2}{2} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}.$$

You get a common denominator by multiplying by $\frac{a}{a}$, where a varies based on what you need in each case to make a common denominator. Subtracting is similar.

$$\frac{4}{x} - \frac{y}{5} = \frac{4}{x} \cdot \frac{5}{5} - \frac{y}{5} \cdot \frac{x}{x} = \frac{20}{5x} - \frac{xy}{5x} = \frac{20 - xy}{5x}$$

Now a little practice.

(1) $\frac{3}{5} + \frac{4}{x}$

(2) $\frac{7}{x+2} - 8$

(3) $1 + \frac{1}{y} - \frac{1}{z}$

2. Multiplying Fractions

Multiplying fractions is so much easier. Just multiply across.

$$\frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56}$$

Yes, after multiplying you can sometimes reduce the result to get a fraction in "lowest terms." We'll talk about cancellation later on. First, just practice multiplying without simplifying.

(4) $\dfrac{3}{5} \cdot \dfrac{4}{x}$

(5) $\dfrac{7}{x+2} \cdot 8$

(6) $\dfrac{3x}{7z} \cdot \dfrac{1}{x^2} \cdot 4$

3. Dividing with Fractions

Multiply by the reciprocal of the denominator. Add "$\overline{1}$" as needed. Here are three examples:

$$\frac{\frac{3}{8}}{\frac{2}{7}} = \frac{3}{8} \cdot \frac{7}{2} = \frac{21}{16}$$
$$\frac{\frac{x}{5}}{3} = \frac{\frac{x}{5}}{\frac{3}{1}} = \frac{x}{5} \cdot \frac{1}{3} = \frac{x}{15}$$
$$\frac{z}{\frac{3}{4}} = z \cdot \frac{4}{3} = \frac{z}{1} \cdot \frac{4}{3} = \frac{4z}{3}$$

Practice!

(7) $\frac{\frac{3}{5}}{\frac{4}{x}}$

(8) $\dfrac{7}{x+2} \div 8$

(9) $\frac{\frac{1}{x}}{\frac{1}{x}}$

(10) $\dfrac{3x}{7z} \div (\dfrac{1}{x^2} \div 4)$

(11) Here's a good question for you: is $\dfrac{\frac{a}{b}}{c}$ the same as $\dfrac{a}{\frac{b}{c}}$?

4. Canceling in Fractions

One of the biggest pitfalls to watch out for! Remember: you can only cancel in a fraction if you cancel from *every* term on top and *every* term on bottom at the same time! Some fractions are simple and only have one term on top and bottom, like this one:

$$\frac{4x}{17x} = \frac{4}{17}$$

Others might have multiple terms on top and bottom. In the following example, notice how we cancel a 2 from *every* term on top (4 and $8x$) and *every* term on bottom (which just the 2 in this case).

$$\frac{4+8x}{2} = \frac{2+4x}{1} = 2+4x$$

Now here's a BAD example. DO NOT do this!

$$\frac{x+6}{3} = \frac{x+2}{1} = x+2$$

What went wrong? We canceled from the 6 and the 3, but not from the x. You can't cancel unless you cancel from *every* term! Canceling can get fancy. For example, suppose we had something like

$$\frac{4+8x}{1+2x}.$$

We can't cancel an x from this (since there's no x to cancel out of the 4 on the top or the 1 on the bottom). We can't cancel a 2 out (since there's no 2 to cancel from the 1 on the bottom). But, we *can* factor and cancel.

$$\frac{4+8x}{1+2x} = \frac{4(1+2x)}{1+2x} = \frac{4}{1} = 4$$

What we did is cancel a "$1+2x$" from every term on top and bottom. Factoring and canceling is common. In the following exercises, cancel as much as possible. If it's not possible to cancel anything, say so.

(12) $\dfrac{3x^2+x}{x}$

(13) $\dfrac{3+6x}{x}$

(14) $\dfrac{3+6x}{3}$

(15) $\dfrac{3+6x}{1+x}$

(16) $\dfrac{3+6x}{1+2x}$

(17) $\dfrac{3+6x}{3-6x}$

(18) $\dfrac{3+6x}{-3-6x}$

(19) $\dfrac{x+y}{y}$

(20) $\dfrac{b}{b-a}$

(21) $\dfrac{3a^2-2a}{xa}$

(22) $\dfrac{4y-8}{y-2}$

(23) $\dfrac{x-2}{x^2-4}$

5. Summary

There are reasons for all of these rules – very good reasons that make sense. They're not magic. For example, why do we need a common denominator to add fractions? Think about what $\frac{1}{2}$ means. It means half of a whole. Say half a dollar. Then $\frac{1}{4}$ means a quarter of a dollar, which we call, very creatively, a quarter. Then what is $\frac{1}{2}+\frac{1}{4}$? If we add straight across, we would think it was $\frac{2}{6}=\frac{1}{3}$ of a dollar, which it clearly is not. If we just add the tops without getting a common denominator, we would think it was $\frac{2}{2}$ or $\frac{2}{4}$ or something, which it clearly is not. Only by getting a common denominator and adding the numerators and finding that it is $\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$ does it make sense, a half dollar plus a quarter is $\frac{3}{4}$ of a dollar, or 75 cents. Multiplication makes just as much sense, as does division, but this is review and not really the place to go into details. If you do want some more explanation, try the Khan Academy explanation of adding/subtracting fractions or multiplying/dividing fractions.

You've practiced all of these techniques one at a time. Can you combine them?

(24) Simplify $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$.

(25) Simplify $\dfrac{1}{x+2} - \dfrac{1}{x-1}$.

(26) Simplify $\dfrac{x + \frac{1}{x-2}}{x}$.

(27) Simplify $\dfrac{2}{x} + \dfrac{1}{(x-1)^2}$.

(28) Simplify $\dfrac{x}{y} + \dfrac{y}{z} + \dfrac{z}{x}$.

(29) Simplify $\dfrac{a-b}{\frac{a}{b} - \frac{b}{a}}$.

(30) True or false? $\dfrac{x}{x+y} = 1 + \dfrac{x}{y}$.

(31) True or false? $\dfrac{x+y}{x} = 1 + \dfrac{y}{x}$.